Worksheet: Binomial Distribution

Multiple Choice
Identify the letter of the choice that best completes the statement or answers the question.

____ 1. Which of the following is not a property of a Binomial Experiment?
   a. All trials are identical.
   b. Each trial has only two possible outcomes.
   c. The probability of success may change from trial to trial.
   d. The purpose of the experiment is to determine the number of successes that occurs during the *n* trials.

____ 2. In the expression $\binom{8}{3}(0.2)^3(0.8)^5$, which value represents the number of trials?
   a. 2   c. 5
   b. 3   d. 8

____ 3. In the expression $\binom{7}{2}(0.4)^2(0.6)^5$, which value represents the probability of failure?
   a. 0.6   c. $(0.4)^2$
   b. 0.4   d. $(0.6)^5$

____ 4. In the expression $\binom{10}{3}(0.5)^3(0.5)^7$, which value represents the number of successes?
   a. 3   c. 5
   b. 10   d. 7

____ 5. Which expression describes the probability of *k* "3s" being rolled on 20 successive rolls of a six-sided die?

   a. $\binom{20}{k}\left(\frac{1}{6}\right)^k\left(\frac{5}{6}\right)^{20-k}$ c. $\binom{20}{k}\left(\frac{3}{6}\right)^k\left(\frac{3}{6}\right)^{20-k}$

   b. $\binom{20}{k}\left(\frac{5}{6}\right)^k\left(\frac{1}{6}\right)^{20-k}$ d. $\binom{20}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{17}$

____ 6. The probability of a computer memory chip being defective is 0.02. Which of the following statements is true?
   a. In a shipment of 100 chips, two will be defective.
   b. The expected number of defective chips in a shipment of 500 is ten.
   c. In a shipment of 1000 chips, it is certain that at least one will be defective.
   d. All statements above are false.

____ 7. A young couple plans to have a family with four children. Assuming that the behaviour of their first child does not cause them to alter their plans, what is the expected number of girls for their family?
   a. 2.5   c. 2
   b. 2.25   d. 1.5

Short Answer

8. A hockey goaltender has a save percentage of 0.920. This means that the probability of any single shot taken on the goaltender being a goal is 0.08. What would be the expected number of goals scored on this goaltender in a game where she faced 35 shots?

9. A manufacturer of halogen bulbs knows that 3% of the production of their 100 W bulbs will be defective. What is the probability that exactly 5 bulbs in a carton of 144 bulbs will be defective?

10. A fair die has four faces numbered one to four. What is the probability of rolling a two exactly three times in ten rolls of the die?

11. A packet of carrot seeds has a germination rate of 92%. In other words, the probability of any seed sprouting is 0.92. How many seedlings would you expect in a row of 50 seeds?

12. A packet of vegetable seeds has a germination rate of 96%. What is the probability that exactly 10 of 12 seeds planted will sprout?

Problem

13. A student writes a five question multiple-choice quiz. Each question has four possible responses. The student guesses at random for each question. Calculate the probability for each possible score on the test from 0 to 5.

14. There are 10 members on a committee. The probability of any member attending a randomly chosen meeting is 0.9. The committee cannot do business if more than 3 members are absent. What is the probability that 7 or more members will be present on a given date?

15. A salesman has a 20% probability of making a sale to any customer who enters his department. On a typical day, he will meet 30 customers. What minimum number of sales will he have an 88% certainty of making for any given day?

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Answer Section

MULTIPLE CHOICE

1. ANS: C
2. ANS: D
3. ANS: A
4. ANS: A
5. ANS: A
6. ANS: B
7. ANS: C

SHORT ANSWER

8. ANS:
    2.8 goals
9. ANS:
    0.169
10. ANS:
    0.2503
11. ANS:
    46
12. ANS:
    0.0702

PROBLEM

13. ANS:

$$P(X = 0) = \binom{5}{0}\left(\frac{1}{4}\right)^{0}\left(\frac{3}{4}\right)^{5} = 0.2373$$

$$P(X = 1) = \binom{5}{1}\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{4} = 0.3955$$

$$P(X = 2) = \binom{5}{2}\left(\frac{1}{4}\right)^{2}\left(\frac{3}{4}\right)^{3} = 0.2637$$

$$P(X = 3) = \binom{5}{3}\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{2} = 0.0879$$

$$P(X = 4) = \binom{5}{4}\left(\frac{1}{4}\right)^{4}\left(\frac{3}{4}\right)^{1} = 0.0146$$

$$P(X = 5) = \binom{5}{5}\left(\frac{1}{4}\right)^{5}\left(\frac{3}{4}\right)^{0} = 0.0010$$

14. ANS:
The total probability is the sum of the probabilities for having 7, 8, 9 or 10 members present.

$$P(X = 7) = \binom{10}{7}(0.9)^{7}(0.1)^{3} = 0.0574$$

$$P(X = 8) = \binom{10}{8}(0.9)^{8}(0.1)^{2} = 0.1937$$

$$P(X = 9) = \binom{10}{9}(0.9)^{9}(0.1)^{1} = 0.3874$$

$$P(X = 10) = \binom{10}{10}(0.9)^{10}(0.1)^{0} = 0.3487$$

The total probability is 0.9872.

15. ANS:
Let X represent the number of sales made. Calculate the probability for each value of X starting at 0. Once we have a total probability of 10% for the first n values, then we will have a 90% probability that $X > n$.

$$P(X = 0) = \binom{30}{0}(0.2)^{0}(0.8)^{30} = 0.0012$$

$$P(X = 1) = \binom{30}{1}(0.2)^{1}(0.8)^{29} = 0.0093$$

$$P(X=2) = \begin{pmatrix} 30 \\ 2 \end{pmatrix} (0.2)^2 (0.8)^{28} = 0.0337$$

$$P(X=3) = \begin{pmatrix} 30 \\ 3 \end{pmatrix} (0.2)^3 (0.8)^{27} = 0.0785$$

The salesman will make 3 or fewer sales on 12.3% of the days he works. He would expect to make 4 or more sales on the remaining 87.7% of the days he works.